Title: Empirical Analysis of Short and Long-Run Equilibrium Relationships between Exchange Rates, CPI and Monetary Policy in the Ghanaian Economy

Executive Summary

This project examines the long-run relationships and short-term dynamics among key Ghanaian macroeconomic indicators: Consumer Price Index (CPI), USD/GHS exchange rate, and Monetary Policy Rate (MPR), using monthly data from January 2014 to May 2025. The analysis tests for non-stationarity, explores cointegration, and estimates a Vector Error Correction Model (VECM). Results show that inflation and exchange rates are closely linked in the long run, while monetary policy actively responds to deviations from equilibrium, highlighting the interplay of these variables across different time horizons.

Data Preprocessing

The CPI data was obtained from the Ghana Statistical Service StatsBank website. Exchange rate and MPR data were also sourced from the Bank of Ghana website and monthly reports. All variables were compiled at a monthly frequency between January 2014 and May 2025.

Daily exchange rates were converted to monthly averages. Natural logarithms were applied to CPI and USD/GHS to stabilise variance and to allow interpretations in percentage terms:

$$\mathrm{logCPI}_t = \log\left(\mathrm{CPI}_t\right), \mathrm{logUSDGHS}_t = \log\left(\mathrm{USDGHS}_t\right)$$

MPR was used in levels. The analysis considers the vector as:

$$Y_t = \begin{bmatrix} \log\left(\mathrm{CPI}_t\right) \\ \log\left(\mathrm{USDGHS}_t\right) \\ \mathrm{MPR}_t \end{bmatrix}$$

Methodology

1. Stationarity Check: Stationarity was assessed using the Augmented Dickey-Fuller (ADF) test:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t$$

where Δy_t is the first difference, t is time, p is the lag order, and ε_t is the white noise term.

2. Cointegration Testing: Johansen's trace test was used to assess the existence of cointegrating relationships in a VAR system, reformulated as a Vector Error Correction Model (VECM):

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t$$

where $\Pi = \alpha \beta^{\top}$. β contains the cointegrating vectors, and α the adjustment coefficients.

3. Lag Selection:

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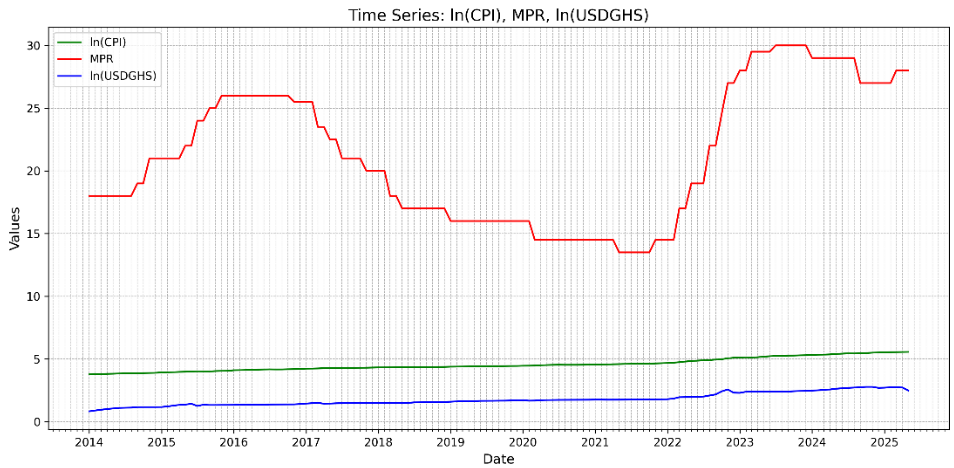
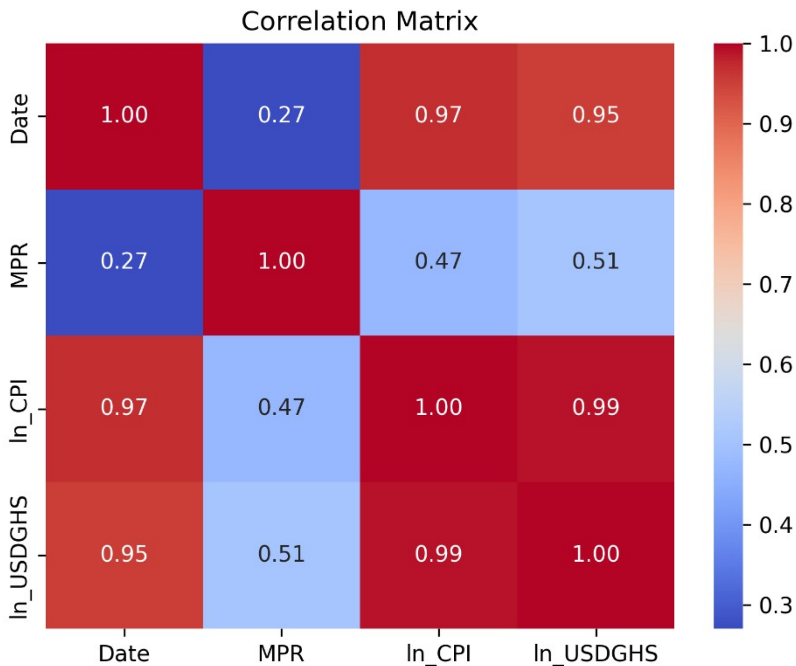
	AIC	BIC	FPE	HQIC
0	-8.200	-8.067	0.0002746	-8.146
1	-15.62	-15.29*	1.647e-07	-15.48
2	-15.76	-15.23	1.428e-07	-15.55*
3	-15.82	-15.09	1.343e-07	-15.53
4	-15.87	-14.94	1.280e-07	-15.50
5	-15.87	-14.74	1.293e-07	-15.41
6	-15.82	-14.49	1.356e-07	-15.28
7	-15.93*	-14.40	1.226e-07*	-15.30

VAR models were estimated using lags 0 to 7. Based on a balance between AIC, BIC, FPE and HQIC criteria, a lag order of 2 was selected for the VAR, corresponding to one lag difference in the VECM.

Exploratory Data Analysis





Results And Discussion

Table 1: ADF Unit Root Test Results

Variable	ADF Statistic	p-value
MPR	-2.2804	0.1784
log(CPI)	0.3928	0.9812
log(USD/GHS)	-1.2080	0.6701

Table 2: Johansen Trace Test Results

Rank	Trace Stat	CV (90%)	CV (95%)	CV (99%)
$r = 0$	31.15	27.07	29.80	35.46
$r \leq 1$	6.19	13.43	15.49	19.93
$r \leq 2$	0.54	2.71	3.84	6.63

From table 1, all variables have p-values above 0.05, indicating that they are non-stationary in levels. This justifies cointegration analysis.

Table 2; Only the null hypothesis of no cointegration is rejected, indicating one cointegrating vector among the variables.

Short-Run Dynamics

Short-run dynamics for the indicators are shown in Table 3 to 5

Table 3: Short-run dynamics for ln_CPI

Variable	Coefficient	z-stat	p-value
L1.ln_CPI	0.3278	4.315	0.000
L1.MPR	0.0041	2.953	0.003
L1.ln_USDGHS	0.0901	4.294	0.000

Table 4: Short-run dynamics for MPR

Variable	Coefficient	z-stat	p-value
L1.ln_CPI	18.4825	4.008	0.000
L1.MPR	-0.0564	-0.676	0.499
L1.ln_USDGHS	3.0592	2.403	0.016

Table 5: Short-run dynamics for ln_USDGHS

Variable	Coefficient	z-stat	p-value
L1.ln_CPI	-0.5075	-1.491	0.136
L1.MPR	0.0047	0.755	0.450
L1.ln_USDGHS	0.1002	1.066	0.286

From the table 3, we observed that the inflation series is persistent and all the lagged terms are statistically significant, showing that past CPI, MPR and exchange rate affect current inflation.

In table 4, the results indicate that MPR responds positively to lagged inflation and to movements in the exchange rate however, lagged value of MPR itself is not significant, suggesting that the policy rate is actively managed rather than following a strongly autoregressive pattern.

While in table 5, the results show that none of the lagged variables are significant predictors of short-run changes in the exchange rate, suggesting that it is largely driven by external factors rather than domestic variables in the short term.

Long-Run Relationship

Cointegration relations for loading-coefficients-column 1

	coef	std err	z	P>|z|	[0.025	0.975]
beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	0.0102	0.005	1.893	0.058	-0.000	0.021
beta.3	-1.0055	0.061	-16.520	0.000	-1.125	-0.886
const	-2.7457	0.113	-24.271	0.000	-2.967	-2.524

The estimated cointegrating relationship is:

$$\log\backslash(\mathrm{CPI}\backslash) = -0.0102 \cdot \mathrm{MPR} + 1.0055 \cdot \log\backslash(\mathrm{USDGHS}\backslash) + 2.7457$$

- The coefficient on log(USDGHS) is positive and significant, suggesting that currency depreciation is linked to higher price levels.
- The coefficient on MPR is negative, indicating that higher interest rates are associated with lower inflation, though the significance is marginal.

This implies that, in the long run, a 1% increase in ln_USDGHS is associated with approximately a 1% increase in ln_CPI. The MPR has a small negative long-run effect on inflation although its eco nomic magnitude is limited.

Adjustment Coefficients (α):

The error-correction terms indicate how each variable adjusts to deviations from the long-run equi librium. Table 6 summarizes these estimates.

Table 6: Error-Correction Coefficients

Variable	α (ec1)	Std. Err.	z-stat	p-value
lnCPI	0.0281	0.005	5.366	0.000
MPR	-0.9105	0.318	-2.865	0.004
lnUSDGHS	0.0754	0.023	3.217	0.001

The results show that ln_CPI adjusts gradually, correcting about 2.8% of disequilibrium per month while MPR adjusts strongly and significantly, indicating an active policy response to deviations from equilibrium. The exchange rate also adjusts significantly but at a moderate pace.

DIAGNOSTIC PLOTS

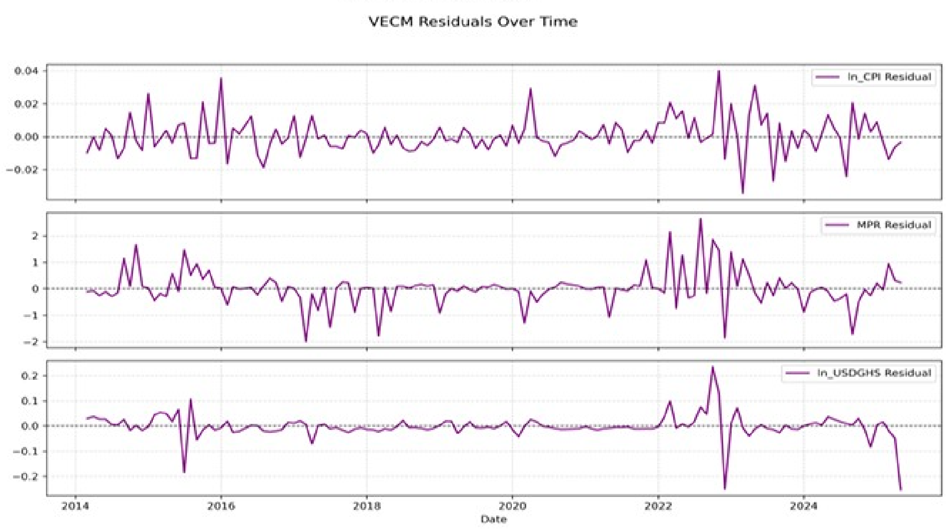


Figure 1: Residual plot

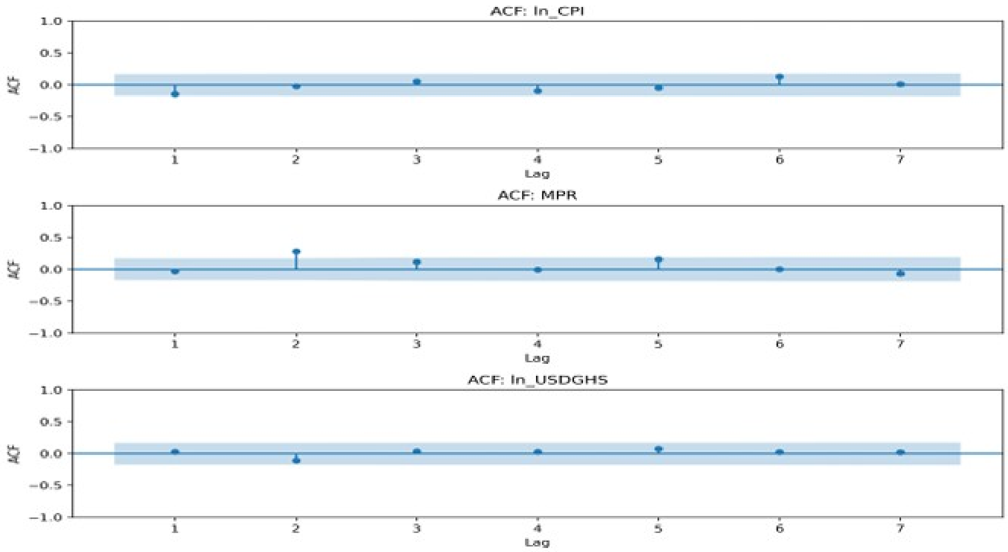


Figure 2: Autocorrelation Function plot

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Ljung-Box Test Results for ln_CPI:
Variable  Lag  lb_stat  lb_pvalue
  ln_CPI    1   2.8680     0.0904
  ln_CPI    2   2.9540     0.2283
  ln_CPI    3   3.3627     0.3390
  ln_CPI    4   4.6555     0.3245
  ln_CPI    5   4.9915     0.4169
  ln_CPI    6   7.4934     0.2776
  ln_CPI    7   7.5092     0.3779

Ljung-Box Test Results for MPR:
Variable  Lag  lb_stat  lb_pvalue
     MPR    1   0.1208     0.7282
     MPR    2  10.8372     0.0044
     MPR    3  12.7308     0.0053
     MPR    4  12.7353     0.0126
     MPR    5  16.1916     0.0063
     MPR    6  16.1922     0.0128
     MPR    7  16.8570     0.0183

Ljung-Box Test Results for ln_USDGHS:
 Variable  Lag  lb_stat  lb_pvalue
ln_USDGHS    1   0.1023     0.7491
ln_USDGHS    2   1.6604     0.4360
ln_USDGHS    3   1.8485     0.6044
ln_USDGHS    4   1.9507     0.7448
ln_USDGHS    5   2.7836     0.7333
ln_USDGHS    6   2.8683     0.8252
ln_USDGHS    7   2.9232     0.8920
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Figure 3: Ljung-Box Test

From figure 1, the residuals of In_CPI are centered around zero with low variance but slightly increasing variance around 2022 onward and that of USDGHS are mostly stable over time but there are a few sharp spikes between 2022 and 2023 likely reflecting volatility during the period, while MPR have high residual variance with frequent spikes suggesting potential outliers or structural breaks

In figure 2, all the ln_CPI and USDGHS Residue spikes are within the 95% confidence bands across the lags 0 to 7, confirming the absence of significant autocorrelation and white noise residuals while that of MPR, there is a visible spike at lag 2 which exceeds the confidence bounds, indicating residual autocorrelation.

In figure 3 also, all Ljung-Box p-values for ln_CPI and USDGHS Residues lags from 1 through 7 are greater 0.05, indicating that no significant autocorrelation detected, confirming the residuals are consistent with white noise. While that of MPR Residue, shows significant autocorrelation from lag 2 onwards with p-values below 0.05, indicating that the model fails to fully capture the time dynamics of the MPR variable.

Economic Implications

The analysis indicates that inflation and the exchange rate are strongly linked in the long run, with an estimated elasticity close to one. MPR acts as the primary adjustment mechanism, responding to deviations from equilibrium. In the short run, inflation dynamics show significant exchange rate pass-through and persistence. The exchange rate however, appears relatively exogenous in short-run dynamics, suggesting it is influenced more by external factors.

Model Pitfalls and Deployment

Analysis of the estimated parameters and diagnostic plots revealed that the Monetary Policy Rate (MPR) residuals exhibit strong autocorrelation and large spikes, particularly after 2022. This indicates that the model does not fully capture MPR dynamics, likely due to outliers, structural breaks, or insufficient lags. Consequently, the model is suitable for forecasting the Consumer Price Index (CPI) and USD/GHS exchange rate, and the long-run relationships remain informative. However, short-term MPR forecasts are unreliable until these issues are addressed.

Conclusion

The analysis confirms a stable long-run relationship between inflation and the exchange rate in Ghana. Monetary policy responds significantly to deviations from equilibrium, underscoring its role in macroeconomic stabilization. Short-run dynamics show notable interactions between inflation and the exchange rate, while exchange rate movements are less influenced by domestic variables. Overall, the findings emphasize the importance of effective monetary policy in maintaining macroeconomic stability.